Using physical experiments in mathematics lessons to introduce mathematical concepts

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Abstract
Physical experiments have a great potential in mathematics lessons. Students can actively discover how mathematical concepts are used. This paper shows results of research done how students got to know the different aspects of the concept of variable by doing simple physical experiments. Further it will be shown what other concepts could be touched by the same treatment.

Background

The project ScienceMath is an interdisciplinary European co-operation project for the promotion of mathematical and scientific literacy. The basic idea is to encourage mathematic learning in scientific contexts and activities of the pupils. The pupils shall experience Mathematics in an appropriate interesting and important way by the means of extra-mathematical references. With the aid of scientific contexts and methods the gap between formal mathematics and authentic experience shall be closed and on the other hand the variety of mathematic items shall be experienced. While doing experiments different mathematical concepts and methods are used, especially the concept of function and the concept of variable.

Experiments to introduce the concept of variable

Malle (1986) differentiates three aspects of variable. If a variable stands for an unknown item or unknown object it is considered as variable as an object. The placeholder aspect means variables can be substituted through a number. If variables stand for meaningless symbols, with which certain rules can be applied, he speaks of the calculation aspect. He further differentiates variable as an object into single number aspect and interval aspect. Single number aspect means an arbitrary but fixed number within a given domain. Variables which match to the interval aspect represent the whole domain. Within that interval aspect it can be differentiated between simultaneous aspect (representation at the same time) or changing aspect (representation in succession). Comparing the decomposition of the concept of variable according to Trigueros et. al (1996) into generalized number (representing a general entity, which can assume any value), specific unknown (representing a constant value, which might change in another situation) and variable in a functional relationship, generalized number can be attributed to variable as an object, which can be represented at the same time or in succession. Specific unknown is equivalent to the static component. To conceptualize variables in a functional relationship, knowledge of dynamic and static components is needed.

Regarding quantitative experiments a lot of the aspects mentioned above can be touched. Measuring values are unknown objects representing an interval of numbers. They may change causing changes of other related measuring values. The relationship between measuring values is given by specific unknowns. They remain constant in the same situation and might change if settings or the environment change. Since "students' conceptions of a mathematical concept is determined by the set of specific domains in which that concept has been introduced for the student" (Michelsen, 2006), experiments have a great potential to introduce the concept of variable.

Using physical experiments to introduce mathematical concepts means putting emphasis on the mathematical aspects. That means some physical aspects should play a minor role. For example the way measuring instruments work, why an experiment is set up this way or another are not major concerns of mathematics. Major concern of mathematics is the reliability of the measuring values involved to do mathematics. Even then measurement errors occur, i.e. functional relationships between measuring values are ideal models and reflect reality only if one takes these errors in consideration. To minimize physical aspects which might trouble students, experiments should be easy to handle. Students should be acquainted to measuring devices. If they are not, they should be easy to understand then. Especially if examining inversely proportional relationships the measuring interval should be well chosen since there are intervals, in which slight changes of one measuring value cause a big change of the other. Therefore the variance of the products increases, making it hard to see that these products are constant.

Study design

Which aspects of the concept of variable and how these were touched was examined in three different schools with a total number of 60 students. 28 of them (S1-S18 and S51-S60) were

6th graders of a German Gymnasium and 32 were students of a German Realschule of grade seven. They were invited to come to school an extra afternoon and were doing a physical experiment in pairs and filling out a worksheet and were interviewed afterwards by students of the University of Education and the author. Problem oriented interviews were chosen, since they allow an open atmosphere, wherein the students could speak freely within a given framework (Lamnek, 2005). Interviewers gave guiding open questions only. Even if students came voluntarily, both stronger and less strong students were part of the study. Three different experiments were used: buoyancy, thermal expansion and Boyle's law. In the buoyancy experiment students measure gravitational forces of a mass in air and in water and have to find a proportional relationship between theses forces. Because of thermal expansion the height of a pillar of liquid will increase if the temperature increases. In that case difference of height is proportional to the difference of temperature. Boyle's law describes an inversely proportional relationship between pressure and volume at a given temperature. In the experiment students measure pressure and volume of a given piston. The worksheets asked all the aspects of the concept of variable in a descriptive and a formal way using open tasks if possible. They started with an impulse from everyday life to encourage students and to make connections to the experiment. Then they shall experience the functional relationship qualitatively, i.e. change of one measuring value causes change of other measuring value and how they change. After measuring at least six pairs they were asked to come up with a formula describing the phenomenon. Then questions concerning possible values are asked followed by a question how changes done at the setting or the environment might change the functional relationship. To touch the specific unknown explicitly they were asked to set up a more general formula which would be valid in different situations, too . At the end they should write down what they have learned to a student who hasn't seen such a formula. The design allows students to work by themselves. Instructions were only given, when students were at a loss and if tasks were essential for the following tasks.

Aspects of variable touched by the experiments

Students are able to identify the measurands with their chosen variables

buoyancy experiment

S37 [1:35]: uhm, what was our formula?
S36 [1:38]: L divided by W equals 1.1.
S35 [1:40]: and translated: air (German: Luft) divided by water (German: Wasser) equals 1.1.
[1:42]: (S35, S36, S37 laugh)
S35 [1:46]: correct.

Boyle's law

I1 [7:52]: What are these cm stand for?
S1 [7:55]: Mm, if you look at that scale, there is 6 cm.
I1 [7:57]: Mmhmm.
S1 [7:58]: So for the respective number.
I1 [8:00]: And the x?
S1 [8:02]: Stands for the respective pressure.

Variables make sense to them. Most of the time they used words or the first letter of the measuring values involved. The 7th graders who had had a formal introduction to the concept of variable in class a few weeks before the study often chose x and y as their variables.

All possible values of the measurand depend on the setting. When students were asked which values were represented by their variables, most of them wrote the interval they have measured. A few students extrapolated to all positive numbers. Considering functional relationships students on the one side experienced it statically like the students above, on the other side students touched the functional relationship dynamically. This aspect was very dominant.

Thermal expansion experiment

S60 [10:03]: Well, if the liquid raises, then the heat, that means the temperature has had to increase as well, otherwise the liquid doesn't change. This we have seen, once we added warm water, the pillar of liquid went higher immediately.

Boyle's Law experiment

S56 [2:42]: So, if you turn the piston, and for example the number of cm increases, then the pressures decreases.

Students are also able to explain, how the functional relationship can change due to a different setting. Both qualitative and quantitative explanations are given.

Thermal expansion

S10 [3:21]: that, if the glass tube is thicker, then it raises slower and if the glass tube is thinner, the liquid raises quicker.

Boyle's Law experiment

S42 [4:45]: Yes, it could be possible, that at position 5cm, the pressure now would be 2 or 1, depending how much air is in it.

Buoyancy experiment

I2 [10:52]: What changes at your given formula, if there is sea water?

S5 [11:06]: Well, then uhm it will be air divided by water with salt, too and that is equal to an unknown number.

Thermal expansion experiment

S17 [5:53]: The x is the constant, in our case it was 6 and that x can also change, if the pillar of liquid is smaller or thicker.

Therefore it is a good basis for a sophisticated discussion in class in which less strong students can join the discussion, too.

Possible obstacles

There are two different kinds of obstacles, inner mathematical obstacles and such caused by physics. Considering inner mathematical ones, especially weaker students had big problems finding a formula describing the phenomenon and got demotivated. Since most of them touched the different aspects of variable on a descriptive level, a worksheet asking on that level only, would be more appropriate. The introduction of variables would be afterwards then. Quite a few students from the second group, who got introduced to the concept of variable in advance, thought finding a formula had to be something with x. Students from the two other schools though, who hadn't had an introduction, chose more natural names as their variables. Two main obstacles caused by physics troubled the students. First some students had problems using the physical material. They didn't dare touching it or didn't know what to do with it. This was only some students though; other students were very curious trying out new material. Talking with the science teacher of a class about such possible problems would probably help to minimize or get rid of this obstacle. The second obstacle troubled most of the students, measurement errors. Due to measurement errors students' quotients weren't the same and long decimal numbers, but close to each other. Tough they got an introduction about measurement errors and how to handle those, they still had problems. So they were told to consider measurement errors a second time, when they were trying to find the functional relationship. Once they accepted these errors, then they were convinced about their formula found.

Buoyancy experiment

S23 [9:24]: so, I'm 90 percent sure, since all results are similar.

Boyle's Law experiment

S28 [3:26]: Yes, the other values are pretty close. There you see, sometimes it was 6.5 here 6.51 and there 6.75. That was pretty close, that could be equal as well.

This means that a good introduction about measurement errors is essential. Besides that are students more convinced about their result, if they are measuring by their own.

Buoyancy experiment

I2 [4:43]: Mmhmm, if a mathematics teacher said, that your formula would be wrong, are you sure about it anyhow?

S35 [4:53]: Uhm, yes.

S37 [4:54]: Actually yes.

S36 [4:55]: We have done, we have done it by our own.

S37 [4:58]: Yes.

Further potential these experiments have

In the experiments variables are used in a functional relationship. So the same experiment can also be used to introduce the concept of function and especially proportional and inversely proportional functions in addition to the concept of variable. Then the worksheet could be supplemented by a graph to see the relationship between the different representations of a function. Höfer (2008) has shown that experiments have a great potential to promote functional thinking. Besides the concept of function the concept of equivalence can be taught. By giving open tasks different groups can come up with equivalent formulas, all describing the same phenomenon. After the experiments students' results would be a good basis to discuss this principle. Some students even came up with all three equivalent formulas during the treatment.

Thermal expansion experiment

S17 [5:19]: Uhm, we had to find formulas. That was height times x is difference of temperature and difference of temperature divided by x equals height and difference of temperature divided by height equals x then.

Boyle's law experiment

S38 [3:09]: Well, we always multiplied the cm times the air pressure and the result was always about 14 and then it is logical that for example dividing cm by 14 is equal to the air pressure. So it is possible for example to calculate the air pressure without such a device.

Students didn't think about a certain rule. They just found different formulas describing their phenomenon.

Another competency touched by the experiments is modelling. While working on the experiments and the worksheet students go through the whole modelling cycle. By experiments especially the reflection and validation process can be emphasized because measurement errors always occur so that the constant doesn't reflect the measuring values exactly and due to changes of the formula in other setting one has to consider the validity of the formula found. In the interviews students were asked if their formula is valid, since their quotients respectively products weren't exact. It made them think about their model.

Boyle's law experiment

S2 [7:22]: It (measuring) isn't that exact.

I1 [7:27]: But your formula you have written down, is correct, isn't it?

S2 [7:32]: Well, not that exact either. It's like. No it isn't that exact. […] So it could be also 7.1 instead of seven.

I1 [7:48]: mmhmm. Would you say your formula is valid or false?

S2 [7:57]: I would say, the formula is valid, because with this device you cannot determine a value exactly for sure. So my numbers, I have written down, are actually as exact as possibly can be done with this device.

That doesn't mean that results could be falsified.

Buoyancy experiment

S3 [6:59]: And if we did that again and would get other results, we would be in a fix and wouldn't know what would be right.

So experiments have a great potential to go through the modelling cycle and especially promotes the reflection and validation process which can be overseen easily.

Besides the other concepts which could be covered in addition to the concept of variable this interdisciplinary approach could be deepened. This approach presented here used physical contents and methods in mathematics lessons to study mathematical content. Another approach would be if mathematics teacher and physics teacher would teach the same phenomenon at the same time. For example the physics teacher would teach buoyancy using forces in physics class and at the same time the math teacher would use the buoyancy experiment to introduce the concept of variable in mathematics lesson. It would be also possible to teach in common class for a short time to even better see common and subject-specific aspects. An overview of cooperation forms can be found in Beckmann (2009).

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